Titolo del corso: Topics in stochastic Analysis: Nonlinear Filtering and Stochastic Sewing

Docenti: **Katia Colaneri** (Tor Vergata), **Giacomo Lucertini** (University of Augsburg)

Membri del collegio proponente: Stefano Pagliarani, Andrea Pascucci

Ore frontali di lezione: **20 (10 + 10)**

Periodo di lezione: marzo/luglio 2027

Settore/i disciplinare del corso: MAT-06

Tipologia di corso: Base

Modalità di verifica dell'apprendimento: Esame Scritto

Abstract del corso:

Module I (Katia Colaneri): This mini-course introduces the fundamentals of nonlinear filtering for partially observed stochastic systems. We present the derivation and interpretation of the key filtering equations—Zakai and Kushner–Stratonovich—and discuss how conditional distributions of hidden states evolve under noisy observations. The second part focuses on applications to optimization under partial information, with emphasis on financial and economic models where key parameters (e.g., drift, volatility) are not directly observable. Recent literature examples illustrate how filtering techniques integrate with optimal control and decision-making in uncertain environments.

Module II (Giacomo Lucertini): This course explores the Stochastic Sewing Lemma as a tool for studying SDEs with irregular coefficients. While deterministic equations often fail without Lipschitz continuity, additive Gaussian noise, such as Brownian or fractional Brownian motion, can restore well-posedness. We will develop the lemma to obtain sharp estimates for singular drifts where traditional Itô calculus does not apply. These techniques will be used to establish both pathwise and weak uniqueness in non-smooth regimes.

Programma del corso:

Module I:

- Partially observed systems; conditional laws
- Zakai and Kushner–Stratonovich equations
- Filtering in stochastic control
- Examples from finance and economics (drift/volatility estimation, decision rules)

Module II:

- Sewing lemma: deterministic and stochastic versions
- Strong uniqueness through Stochastic Sewing lemma, Brownian noise

- Introduction to Fractional Brownian motion and study of Strong Uniqueness through Stochastic Sewing lemma

- Weak Uniqueness through Stochastic Sewing lemma